The Open University

S207

STANDARD EQUATIONS AND CONSTANTS

This complete list of constants, mathematical formulae and physics equations is included for reference. It may be useful as an aid to your memory but please bear in mind that many of the entries will *not* be needed in this examination.

WEB 96146 9

Useful constants

magnitude of the acceleration due to gravity on Earth	g	=	$9.81\ \mathrm{m\ s^{-2}}$
Newton's universal gravitational constant	G	=	$6.673 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$
Avogadro's constant	N_m	=	$6.022 \times 10^{23}\ \mathrm{mol^{-1}}$
Boltzmann's constant	k	=	$1.381 \times 10^{-23}\ \mathrm{J\ K^{-1}}$
molar gas constant	R	=	$8.314\ \mathrm{J\ K^{-1}\ mol^{-1}}$
permittivity of free space	ε_0	=	$8.854 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$
	$1/4\pi\varepsilon_0$	=	$8.988 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}}$
permeability of free space	μ_0	=	$4\pi \times 10^{-7}\ \mathrm{T\ m\ A^{-1}}$
speed of light in vacuum	c	=	$2.998 \times 10^{8}\ \mathrm{m\ s^{-1}}$
Planck's constant	h	=	$6.626 \times 10^{-34}\ \mathrm{J\ s}$
	$\hbar = h/2\pi$	=	$1.055 \times 10^{-34}\ \mathrm{J\ s}$
Rydberg constant	R	=	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
Bohr radius	a_0	=	$5.292 \times 10^{-11}\ \mathrm{m}$
atomic mass unit	amu (or u)	=	$1.6605 \times 10^{-27}\ \mathrm{kg}$
charge of proton	e	=	$1.602 \times 10^{-19}\ \mathrm{C}$
charge of electron	$-e$	=	$-1.602 \times 10^{-19}\ \mathrm{C}$
electron rest mass	m_e	=	$9.109 \times 10^{-31}\ \mathrm{kg}$
charge to mass ratio of the electron	$-e/m_\mathrm{e}$	=	$-1.759 \times 10^{11}\ \mathrm{C\ kg^{-1}}$
proton rest mass	m_p	=	$1.673 \times 10^{-27}\ \mathrm{kg}$
neutron rest mass	m_n	=	$1.675 \times 10^{-27}\ \mathrm{kg}$
radius of the Earth			$6.378 \times 10^{6}\ \mathrm{m}$
mass of the Earth			$5.977 \times 10^{24}\ \mathrm{kg}$
mass of the Moon			$7.35 \times 10^{22}\ \mathrm{kg}$
mass of the Sun			$1.99 \times 10^{30}\ \mathrm{kg}$
average radius of Earth orbit			$1.50 \times 10^{11}\ \mathrm{m}$
average radius of Moon orbit			$3.84 \times 10^{8}\ \mathrm{m}$

Mathematical formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2π rad = 360°

$\sin(\theta + 2\pi) = \sin(\theta)$

$\cos(\theta + 2\pi) = \cos(\theta)$

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$\boldsymbol{a} \cdot \boldsymbol{b} = ab\cos\theta$

$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$

$\boldsymbol{a} \times \boldsymbol{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$

$|\boldsymbol{a} \times \boldsymbol{b}| = ab\sin\theta$

If $\dfrac{dv}{dt} = \alpha v$, then $v(t) = v_0 e^{\alpha t}$

$e^x e^y = e^{x+y}$

$(e^x)^y = e^{xy}$

$e^{-x} = 1/e^x$

$\log_e e^x = x$

$$\langle x \rangle = \sum_{i=1}^{N} p_i x_i$$

sphere surface area = $4\pi r^2$

sphere volume = $\dfrac{4\pi r^3}{3}$

Derivatives

[A, n, k and ω are constants; x, y and z are functions of t]

x	$\dfrac{dx}{dt}$
A	0
t^n	nt^{n-1}
$\sin(\omega t)$	$\omega\cos(\omega t)$
$\cos(\omega t)$	$-\omega\sin(\omega t)$
e^{kt}	ke^{kt}
Ay	$A\dfrac{dy}{dt}$
$y + z$	$\dfrac{dy}{dt} + \dfrac{dz}{dt}$

Book 2 *Describing motion*

$$s_x = v_x t \quad (v_x = \text{constant})$$

$$s_x = u_x t + \tfrac{1}{2} a_x t^2$$

$$v_x = u_x + a_x t$$

$$v_x^2 = u_x^2 + 2a_x s_x$$

$$s_x = \tfrac{1}{2}(v_x + u_x)t$$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2$$

$$s_{\text{arc}} = R\left|\Delta\theta\right|$$

$$\omega = \left|\frac{\mathrm{d}\theta}{\mathrm{d}t}\right| = \frac{2\pi\,\text{rad}}{T}$$

$$v = \frac{2\pi r}{T} = r\omega$$

$$a = v\omega = r\omega^2 = \frac{v^2}{r}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x(t) = A\sin(\omega t + \phi)$$

$$\frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = -\omega^2 x(t)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e = \frac{1}{a}\sqrt{a^2 - b^2}$$

$$\frac{T^2}{a^3} = K$$

Book 3 *Predicting motion*

$\boldsymbol{F} = m\boldsymbol{a}$

$\boldsymbol{W} = m\boldsymbol{g}$

$$\boldsymbol{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{\boldsymbol{r}}$$

$F_{max} = \mu_{static}N$

$F = \mu_{slide}N$

$F = 6\pi\eta Rv$

$$F = mr\omega^2 = \frac{mv^2}{r}$$

$F_x = -k_s x$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_s}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$E_{trans} = \frac{1}{2}mv^2$

$W = \boldsymbol{F} \cdot \boldsymbol{s}$

$E_{grav} = mgh$

$E_{str} = \frac{1}{2}k_s x^2$

$$E_{grav} = \frac{-Gm_1m_2}{r}$$

$$F_x = \frac{-\mathrm{d}E_{pot}}{\mathrm{d}x}$$

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \boldsymbol{F} \cdot \boldsymbol{v}$$

$x(t) = (A_0 \, \mathrm{e}^{-t/\tau}) \sin(\omega t + \phi)$

$$Q = \frac{2\pi \times \text{total stored energy}}{\text{average energy loss per oscillation}}$$

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\Omega b/m)^2}}$$

$\boldsymbol{p} = m\boldsymbol{v}$

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$$

$\boldsymbol{\Gamma} = \boldsymbol{r} \times \boldsymbol{F}$

$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$

$$\boldsymbol{\alpha} = \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}$$

$$I = \sum_i m_i r_i^2$$

$E_{rot} = \frac{1}{2}I\omega^2$

$E_{kin} = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

$W = \Gamma\Delta\theta$

$P = \boldsymbol{\Gamma} \cdot \boldsymbol{\omega}$

$\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p}$

$\boldsymbol{L} = I\boldsymbol{\omega}$

$$\boldsymbol{\Gamma} = \frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t}$$

Book 4 *Classical physics of matter*

$$\rho = \frac{M}{V} = m \times (\text{number density})$$

$$P = \frac{F_\perp}{A}$$

$$M_\text{m} = M_\text{r} \times 10^{-3}\,\text{kg} = N_\text{m} m$$

$$\alpha = \frac{1}{V}\frac{\text{d}V}{\text{d}T}$$

$$\beta = -\frac{1}{V}\frac{\text{d}V}{\text{d}P}$$

$$PV = nRT = NkT$$

$$U = \frac{f}{2}nRT = \frac{f}{2}NkT$$

$$\langle E_\text{trans} \rangle = \tfrac{3}{2}kT$$

$$p = A\,\text{e}^{-E/kT}$$

$$f(\upsilon) = B\upsilon^2 \exp(-m\upsilon^2/2kT)$$

$$B = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}$$

$$\upsilon_\text{mp} = \sqrt{\frac{2kT}{m}}$$

$$\langle \upsilon \rangle = \sqrt{\frac{8kT}{\pi m}}$$

$$\upsilon_\text{rms} = \sqrt{\frac{3kT}{m}}$$

$$g(E) = C\sqrt{E}\,\text{e}^{-E/kT}$$

$$C = \frac{2}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2}$$

$$E_\text{mp} = \tfrac{1}{2}kT$$

$$\langle E \rangle = \tfrac{3}{2}kT$$

$$\langle E_\text{tot} \rangle = \frac{f}{2}kT$$

$$W = -P\,\Delta V$$

$$\Delta U = Q + W$$

$$C = \frac{Q}{\Delta T}$$

$$C_V = \frac{f}{2}nR$$

$$C_P = C_V + nR$$

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

$$PV = I$$

$$PV^\gamma = A$$

$$\Delta S = \frac{Q_\text{rev}}{T}$$

$$S_2 - S_1 = C_V \log_\text{e}\left(\frac{P_2}{P_1}\right) + C_P \log_\text{e}\left(\frac{V_2}{V_1}\right)$$

$$S = k \log_\text{e} W$$

$$\eta = \frac{W}{Q_\text{h}} = 1 - \frac{T_\text{c}}{T_\text{h}}$$

$$\kappa = \frac{Q_\text{c}}{W} = \frac{T_\text{c}}{T_\text{h} - T_\text{c}}$$

$$P = P_A + \rho g z$$

$$P(z) = P(0)\exp(-z/\lambda)$$

$$\lambda = \frac{kT}{mg}$$

$$\frac{\upsilon_2}{\upsilon_1} = \frac{A_1}{A_2}$$

$$P + \tfrac{1}{2}\rho\upsilon^2 + \rho g h = \text{constant}$$

$$F = \eta A\left|\frac{\Delta\upsilon}{\Delta x}\right|$$

$$Re = \frac{\rho L_0 \upsilon_0}{\eta}$$

Book 5 *Static fields and potentials*

$$\boldsymbol{F}_{\text{grav}} = -\frac{Gm_1m_2}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{F}_{\text{el}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{F}_{\text{grav}}\ (\text{on } m \text{ at } \boldsymbol{r}) = m\boldsymbol{g}(\boldsymbol{r})$$

$$\boldsymbol{F}_{\text{el}}\ (\text{on } q \text{ at } \boldsymbol{r}) = q\boldsymbol{\mathscr{E}}(\boldsymbol{r})$$

$$\boldsymbol{\mathscr{E}}(\boldsymbol{r}) = \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)\hat{\boldsymbol{r}}$$

$$E_{\text{grav}} = -\frac{Gm_1m_2}{r}$$

$$E_{\text{el}} = \frac{q_1q_2}{4\pi\varepsilon_0 r}$$

$$E_{\text{el}}\ (\text{with } q \text{ at } \boldsymbol{r}) = qV(\boldsymbol{r})$$

$$V(\boldsymbol{r}) = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r}$$

$$\mathscr{E}_x = -\frac{\mathrm{d}V(\boldsymbol{r})}{\mathrm{d}x}$$

$$C = \frac{q}{V}$$

$$C = \varepsilon_{\text{r}}\varepsilon_0\frac{A}{d}$$

$$E_{\text{el}} = \frac{qV}{2} = \frac{CV^2}{2} = \frac{q^2}{2C}$$

$$i = \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$i = neAv$$

$$R = \frac{|V_{\text{R}}|}{|i|}$$

$$R = \frac{\rho L}{A}$$

$$V = iR$$

$$R_{\text{eff}} = \sum_i R_i$$

$$\frac{1}{R_{\text{eff}}} = \sum_i \frac{1}{R_i}$$

$$P = iV = i^2R = \frac{V^2}{R}$$

$$V = V_{\text{EMF}} - ir$$

$$q = q_0\,\mathrm{e}^{-t/\tau}$$

$$i = i_0\,\mathrm{e}^{-t/\tau}$$

$$\tau = RC$$

$$\boldsymbol{F}_{\text{m}} = q\,[\boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r})]$$

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

$$B_{\text{centre}} = \frac{\mu_0 Ni}{2R}$$

$$B = \frac{\mu_0 Ni}{l}$$

$$\boldsymbol{F} = q[\boldsymbol{\mathscr{E}}(\boldsymbol{r}) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r})]$$

$$f_{\text{C}} = \frac{1}{2\pi}\frac{|q|B}{m}$$

$$R_{\text{C}} = \frac{m|v_{\text{perp}}|}{|q|B}$$

$$V_{\text{H}} = \frac{i}{nqt}B$$

$$\boldsymbol{F}_{\text{m}} = l[\boldsymbol{i} \times \boldsymbol{B}(\boldsymbol{r})]$$

$$F_{\text{m}} = Bil$$

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\Gamma = iAB$$

Book 6 *Dynamic fields and waves*

$$\phi = AB\cos\theta$$

$$|V_{\text{ind}}(t)| = \left|\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}\right|$$

$$|V_{\text{ind}}(t)| = L\left|\frac{\mathrm{d}i(t)}{\mathrm{d}t}\right|$$

$$L = \frac{\mu A N^2}{l}$$

$$E_{\text{mag}} = \tfrac{1}{2}Li^2$$

$$V_{\text{bat}} - L\frac{\mathrm{d}i(t)}{\mathrm{d}t} = i(t)R$$

$$i(t) = \left(\frac{V_{\text{bat}}}{R}\right)\left(1 - \mathrm{e}^{-Rt/L}\right)$$

$$V_{\max} = i_{\max}\omega L$$

$$V_{\max} = \frac{i_{\max}}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$|V_2(t)| = M\left|\frac{\mathrm{d}i_1(t)}{\mathrm{d}t}\right|$$

$$M = \frac{\mu N_1 N_2 A}{l}$$

$$|V_2(t)| = \frac{N_2}{N_1}\times|V_1(t)|$$

$$y = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

$$v = f\lambda = \frac{\omega}{k}$$

$$v = \sqrt{\frac{F_{\text{T}}}{\mu}}$$

$$f_{\text{obs}} = f_{\text{em}}\left(\frac{v}{v \pm V}\right)$$

$$f_{\text{obs}} = f_{\text{em}}\left(\frac{v \pm V}{v}\right)$$

$$y(x,t) = 2A\cos(\omega t)\sin(kx)$$

$$L = n\left(\frac{\lambda}{2}\right)$$

$$c = f\lambda$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\sin i_{\text{crit}} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$n\lambda = d\sin\theta_n$$

$$\sin\theta = \frac{\lambda}{w}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$m = \frac{v}{u}$$

$$P = 1/f$$

$$P_{\text{total}} = P_1 + P_2 + P_3 + \ldots$$

$$M = \frac{\alpha_{\text{IM}}}{\alpha_{\text{OB}}}$$

$$M = \frac{f_{\text{o}}}{f_{\text{e}}}$$

$$\text{light-gathering power} = \left(\frac{D_{\text{o}}}{D_{\text{p}}}\right)^2$$

maximum light-gathering power

$$= \left(\frac{D_{\text{o}}}{D_{\text{e}}}\right)^2 = \left(\frac{f_{\text{o}}}{f_{\text{e}}}\right)^2$$

$$\text{exposure} = I\Delta t$$

$$F\text{-number} = \frac{f}{D}$$

(***Continued overleaf***)

$$\Delta T = \frac{\Delta T_0}{\sqrt{1 - V^2/c^2}}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$x' = \gamma(x - Vt)$$

$$t' = \gamma(t - Vx/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\upsilon'_x = \frac{\upsilon_x - V}{1 - \frac{V\upsilon_x}{c^2}}$$

$$\boldsymbol{p} = \frac{m\boldsymbol{\upsilon}}{\sqrt{1 - \upsilon^2/c^2}}$$

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\boldsymbol{\upsilon}}{\sqrt{1 - \upsilon^2/c^2}}\right)$$

$$E_{\mathrm{tot}} = \frac{mc^2}{\sqrt{1 - \upsilon^2/c^2}}$$

$$E_{\mathrm{mass}} = mc^2$$

$$E_{\mathrm{trans}} = \frac{mc^2}{\sqrt{1 - \upsilon^2/c^2}} - mc^2$$

$$E_{\mathrm{tot}}^2 = p^2c^2 + m^2c^4$$

$$E = cp$$

Book 7 *Quantum physics: an introduction*

$$E = hf$$

$$\tfrac{1}{2} m_\mathrm{e} \upsilon_\mathrm{max}^2 = hf - \phi$$

$$m_\mathrm{e} \upsilon r = n\hbar$$

$$E_n = -\frac{|E_1|}{n^2}$$

$$\lambda_\mathrm{dB} = \frac{h}{p}$$

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{2m}{\hbar^2}\left(E_\mathrm{tot} - E_\mathrm{pot}(x)\right)\psi = 0$$

$$E_\mathrm{kin} = \frac{\hbar^2 k^2}{2m}$$

$$p = \hbar k$$

$$E_\mathrm{tot} = \frac{n^2 h^2}{8mD^2}$$

$$E_\mathrm{tot} = \frac{h^2}{8mD^2}(n_1^2 + n_2^2 + n_3^2)$$

$$P = |\psi(x_1)|^2 \, \Delta x$$

$$P = |\psi_1(r)|^2 \, \Delta V = |\psi_1(r)|^2 \, 4\pi r^2 \, \Delta r$$

$$E_\mathrm{tot} = -\frac{1}{n^2}\left\{\frac{m_\mathrm{e} e^4}{8h^2\varepsilon_0^2}\right\} = -\frac{13.6\,\mathrm{eV}}{n^2}$$

$$L = \sqrt{l(l+1)}\,\hbar$$

$$L_z = m_l \hbar$$

$$S = \sqrt{s(s+1)}\,\hbar$$

$$S_z = m_s \hbar$$

Book 8 *Quantum physics of matter*

$$G(E) = \frac{2N}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2}\sqrt{E} \times e^{-E/kT}$$

$$D(E) = B\sqrt{E}$$

$$B = \frac{2\pi L^3}{h^3}(2m)^{3/2}$$

$$F_B(E) = \frac{1}{e^{(E-\mu)/kT} - 1}$$

$$F_B(E) = \frac{1}{e^{E/kT} - 1}$$

$$F_F(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$D_p(E) = CE^2$$

$$G_p(E) = CE^2 \times \frac{1}{e^{E/kT} - 1}$$

$$C = 8\pi V/h^3c^3$$

$$N = 2.4C(kT)^3$$

$$U = \frac{\pi^4}{15}C(kT)^4$$

$$P = \frac{U}{3V}$$

$$n = \frac{zN_m}{V_m}$$

$$D_e(E) = B'\sqrt{E}$$

$$B' = \frac{4\pi V}{h^3}(2m_e)^{3/2}$$

$$G_e(E) = B'\sqrt{E} \times \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$E_F = \frac{h^2}{8m_e}\left(\frac{3n}{\pi}\right)^{2/3}$$

$$U = \tfrac{3}{5}NE_F$$

$$P = \tfrac{2}{5}nE_F$$

$$\frac{dN}{dt} = -\lambda N$$

$$N(t) = N_0 \exp(-\lambda t)$$

[END OF BOOKLET]